UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE O Level

MARK SCHEME for the May/June 2006 question paper

4037 ADDITIONAL MATHEMATICS

4037/01 Paper 1 maximum raw mark 80

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the June 2006 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

UNIVERSITY of CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

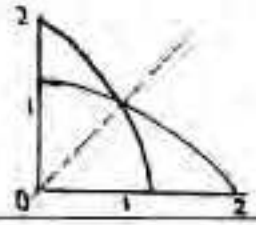
MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW -1,2 This is deducted from A or B marks when essential working is omitted.

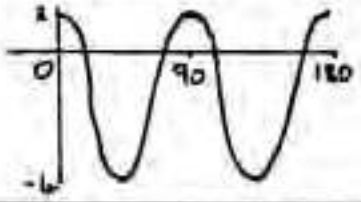
PA -1 This is deducted from A or B marks in the case of premature approximation.

S -1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX -1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1 $y=(x-1)(2x-3)^8$ differential of $(2x-3)^8 = 8(2x-3)^7 \times 2$ → $(x-1)\times$(his dy/dx) + $1\times(2x-3)^8$ If $x=2$, $dy/dx = 17$	B1 M1 M1 A1 [4]	B1 for $8(2x-3)^7$ M1 for $\times 2$ Use of correct formula. Correct only.
2 $y+4x=23$ and $xy+x=20$ → $4x^2-24x+20=0$ $y^2-22y+57=0$ → (5,3) and (1,19) $m=-4$ Perpendicular = ¼ Mid-point = (3,11) → $4y=x+41$	M1 DM1 A1 M1 B1√ A1 [6]	Complete elimination of x or y Soln of quadratic by scheme Use of $m_1m_2=-1$ For his two points Co.
3 (i) Resultant speed = 1000÷2 =500) Correct triangle – 500,150,45° inc  $V^2=500^2+150^2-2.500.150\cos 45$ $V=408$ (ii) $\frac{408}{\sin 45}=\frac{150}{\sin\alpha}$ $\alpha = 15.1°$ wrong triangle – 500,150,45° not included Max B1B0M1M1 3/6	B1 B1 M1 A1 [4] M1 A1 [2]	Anywhere in the question. Any triangle with 45° included between 150 and his resultant. Use of cosine rule. Co. Use of sine rule. Co.
4 (i) $f(x)=p-e^x$ $x=0 \rightarrow p-1=2 \rightarrow p=3$ (ii) $y=0 \rightarrow e^x=3 \rightarrow x=\ln 3$ (iii) 	M1 A1 [2] M1 A1√ [2] B2,1,0 [2]	M for forming equation. Co. M for eqn + logs. Decimals ok. Needs • correct shape. • reflection in y=x or stated • x=1.1 and y=2 marked or implied.

5 (i) $B^2 = \begin{pmatrix}0 & -1\\4 & 3\end{pmatrix}\begin{pmatrix}0 & -1\\4 & 3\end{pmatrix} = \begin{pmatrix}-4 & -3\\12 & 5\end{pmatrix}$ $B^2 - 2A = \begin{pmatrix}-4 & -3\\12 & 5\end{pmatrix} - \begin{pmatrix}-4 & -2\\12 & 4\end{pmatrix}$ $P = \begin{bmatrix}0 & -1\\0 & 1\end{bmatrix}$	M1 M1 A1 [3]	Needs two of the four elements correct Overall method co
(ii) $A^{-1} = \frac{1}{2} \times \begin{pmatrix}2 & 1\\-6 & -2\end{pmatrix}$ $Q = BA^{-1}$ $Q = \begin{pmatrix}0 & -1\\4 & 3\end{pmatrix}\frac{1}{2}\begin{pmatrix}2 & 1\\-6 & -2\end{pmatrix} = \begin{pmatrix}3 & 1\\-5 & -1\end{pmatrix}$	B1 B1 M1 A1 [4]	B1 for ½. B1 for all 4 elements ok The order must be correct. Co.
6 (i) $f(x) = (x+2)(x-(1+\sqrt{3}))(x-(1-\sqrt{3}))$ $= (\ldots)(x^2-2x-2)$ $= x^3-6x-4$	M1 B1 A1 [3]	Realising link of roots & cubic For x^2-2x-2 Co
(ii) Substitute x=3 Remainder = 5	M1 A1√ [2]	Must put x=3 into cubic – not for –3
(iii) $-x = -2 \rightarrow x = 2$ $-x = 1+\sqrt{3} \rightarrow x = -1-\sqrt{3}$ $-x = 1-\sqrt{3} \rightarrow x = -1+\sqrt{3}$ $(-x)^3 - 6(-x) - 4 = -x^3 + 6x - 4$ has the same roots.	M1 A1 [2]	Realising that –x equates with each root or for replacing x by –x and solving the cubic.
7 $v = pt^2 + qt + 4$ t=1 → a = 2pt+q → 2p+q=8 t=2 → $s = \frac{1}{3}pt^3 + \frac{1}{2}qt^2 + 4t$.(+c) → 8p/3 + 2q + 8 = 22 Sim equations → q=5 and p=1½	M1 A1 M1 A1 A1√ DM1 A1 [7]	Using a=dv/dt Co Integrating for s . co. Putting t=2 into his s to form linear eqn in p,q. Soln of equations. Co.

8 (i) $y=\frac{1+\sin x}{\cos x}$ Using quotient rule → $dy/dx=\frac{\cos x\cos x-(1+\sin x)(-\sin x)}{\cos^2 x}$	M1	Correct quotient formula.
	A2,1	Everything ok.
Using $c^2+s^2=1$ → $\frac{1+\sin x}{1-\sin^2 x}$	M1	Using $c^2+s^2=1$ Allow even if u/v incorrect
→ $dy/dx=\frac{1}{1-\sin x}$	A1 ag [5]	Answer given – beware fortuitous answers
(ii) $\int\frac{2}{1-\sin x}dx=(\frac{1+\sin x}{\cos x})\times 2$	M1	Recognition of need to integrate and link with (i) – uses ×2
$\left[\frac{2(1+\sin x)}{\cos x}\right]_{\frac{3\pi}{4}}^{\frac{5\pi}{4}}=-0.828--4.828=4$	M1 A1 [3]	Correct use of limits Co.
9 (a) $u=\log_4 x$ (i) $x=4^u$	B1	
(ii) $\log_4\left(\frac{16}{x}\right)=\log_4 16-\log_4 x$ $=2-u$	M1 A1	Must be "log – log" – or log $(4^2\div 4^u)$ Co
(iii) $\log_x 8=\frac{\log_4 8}{\log_4 x}=\frac{1.5}{u}=\frac{3}{2u}$	M1 A1 [5]	Correct change of base from x to 4 Co
(b) $(\log_3 y)^2+2\log_3 y-8=0$ Solution of quadratic → 2 and – 4 $\text{Log}_3 y=2$ → $y=9$ $\text{Log}_3 y=-4$ → $y=1/81$	B1 M1 A1 A1 [4]	Correct quadratic in $\log_3 y$ Correct method for quadratic Co co
10 $f(x)=3\cos 4x-1$. (i) $\cos 4x=\frac{1}{2}$ (base angle = 70.53) 4x=70.53 or 289.47 or 430.53 or 649.47 x = 17.6° or 72.4° or 107.6° or 162.4°	M1 A1 A1√ [3]	cos4x subject then ÷ by 4 One pair correct. Other pair correct to first answers.
(ii) amplitude = 3	B1	Co
(iii) period = 90^0 or $\frac{1}{2}\pi$	B1	Co
(iv) maximum value = 3-1 = 2 minimum value = -3 –1 = - 4	B1 B1 [4]	Co Co
(v) 	B1 B1√ B1 [3]	• 2 complete cycles • Max "amp – 1" Min "-amp-1" • Starts and finishes correctly

11 EITHER $y = \frac{a}{x+b} \rightarrow xy = -by + a$

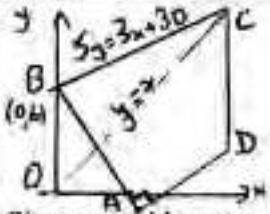
x	0.1	0.4	1.0	2.0	3.0
y	8.0	6.0	4.0	2.6	1.9
xy	0.8	2.4	4.0	5.2	5.7

(i) accurate graph — M1, A2,1 [3] — Knows what to do. Accuracy – to within one small square – by eye.

(ii) $m = -5/4 = -1/b$ $b = 0.8\ (\pm 0.1)$
$C = a/b = 9.0$ $a = 7.2\ (\pm 0.1)$ — M1 A1, M1 A1 [4] — Recognition of $m = -1/b$; Recognition that $\frac{a}{b} = C$

(iii) $x = a/y - b$
Gradient = a
Intercept = -b — M1, A1√, A1√ [3] — Attempt at making x subject. On his a. On his b.

11 OR



(i) Sim eqns with y=x and 5y=3x+30 → (15, 15) — M1, A1 — Uses sim equations. Co.
Gradient of AB = -3/2
Perpendicular = ⅔ — M1 — Uses m_1m_2 – even if ⅔ incorrect
Eqn of AD $y+3=\frac{2}{3}(x-6)$ — M1A1 — Uses $y-k=m(x-h)$ or $y=mx+c$
Put x=15 → y=3 D(15,3) — A1 [6] — Co.

(ii) $AB = \sqrt{(6^2+9^2)}$ $AD = \sqrt{(6^2+9^2)}$ — M1 — One length correct.
Therefore isosceles — A1 — Both correct.

Either Area = ½bh = ½√117√117 = 58.5 — M1A1 [4] — Correct method.

or MV of BD = (7.5,3)
Area = ½BD.MA = ½√234√58.5
= 58.5

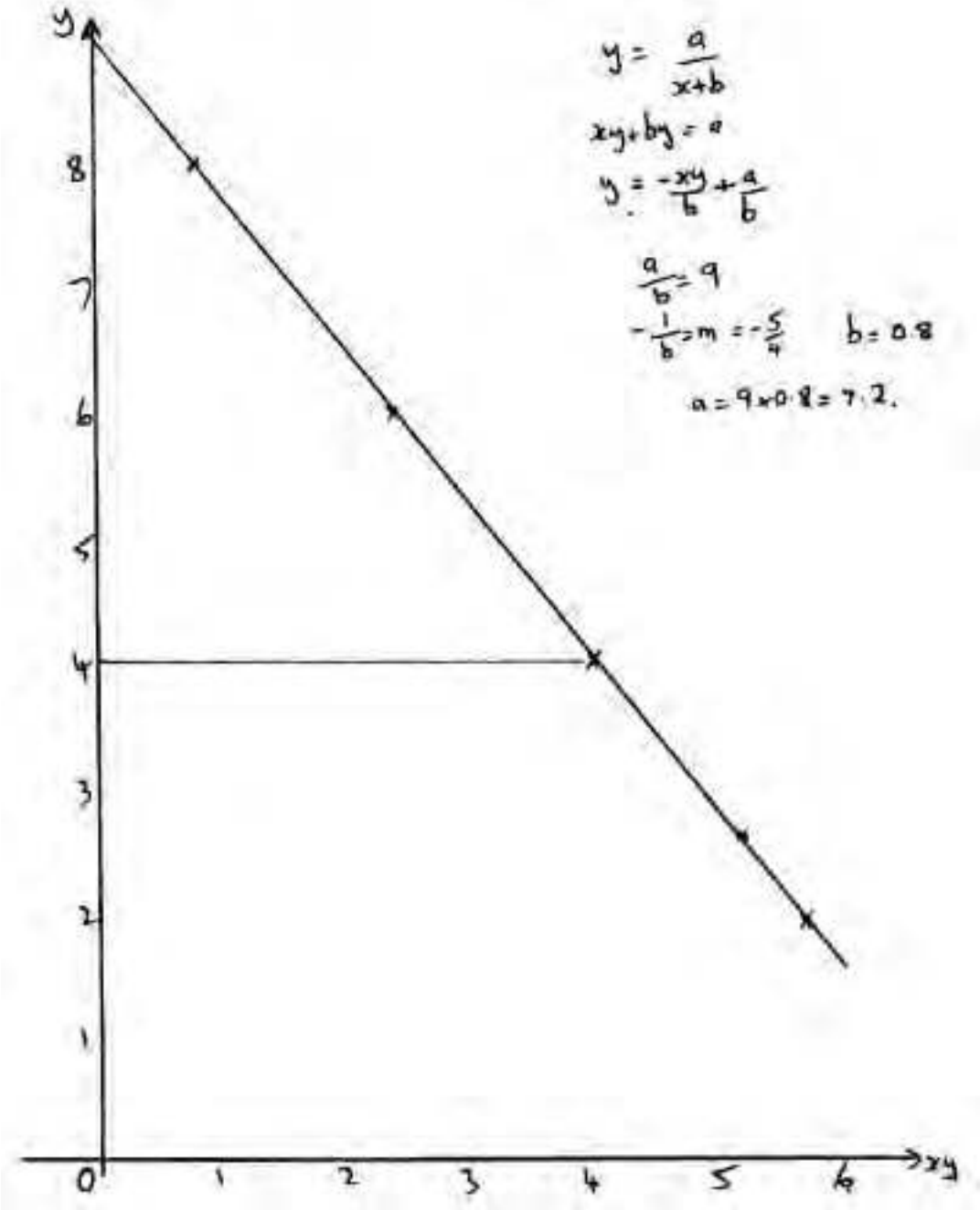
or matrix $\frac{1}{2}\begin{pmatrix} 0 & 15 & 6 & 0 \\ 6 & 3 & -3 & 6 \end{pmatrix}$
$= \frac{1}{2}(-45+36-90-18) = 58.5$

DM1 for quadratic equation. Equation must be set to 0 if using formula or factors.

Formula
Must be correct
– ignore arithmetic and algebraic slips.

Factors
Must attempt to put quadratic into 2 factors.
Each factor then equated to 0.

Q 11 EITHER.



$y = \frac{a}{x+b}$

$xy + by = a$

$y = \frac{-xy}{b} + \frac{a}{b}$

$\frac{a}{b} = 9$

$-\frac{1}{b} = m = -\frac{5}{4}$ $b = 0.8$

$a = 9 \times 0.8 = 7.2.$